

Mesoscopic quantum measurements

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Summary. — The paper discusses dynamics of quantum measurements in mesoscopic solid-state systems. The aim is to show how the general ideas of the quantum measurement theory play out in the realistic models of actual mesoscopic detectors. The two general models of ballistic and tunneling detectors are described and studied quantitatively. Simple transformation cycle demonstrating wavefunction reduction in a mesoscopic qubit is suggested.

1. – Introduction

Despite the long history of quantum mechanics, the quantum measurement problem continues to attract interest motivated mostly by the counter-intuitive features of the “wavefunction reduction”. Although dynamics of any measurement set-up is governed by the Schrödinger equation with an appropriate Hamiltonian, full description of the measurement process can not be obtained without account of the changes in the wavefunction of the measured system caused by the random process of selection of one specific outcome of measurement out of the range of possible outcomes. This selection process is trivial in the case of classical dynamics, when all possible outcomes of measurement are “orthogonal” and the observation of the measured system in one particular state does

not imply any changes in the system beyond the statement that it occupies this and not any other state. For a quantum system, however, existence of the non-commuting observables implies that selection of one particular outcome of measurement can change the wavefunction of the system in a highly non-trivial way. Such a reduction of the wavefunction appears as an evolution principle additional to the Schrödinger equation. Moreover, the changes in the wavefunction of the measured system induced by it can violate the basic features of dynamics which follows from the Schrödinger equation, despite the fact that the measurement process as a whole is governed by this equation. The best known example of this situation is the case of EPR correlations [1] between the two spatially separated spins, which violate the no-action-at-a-distance principle as quantified by the Bell's inequalities [2]. From the perspective of the wavefunction reduction, the EPR correlations appear as a result of selection of one random specific outcome of the local spin measurement. On average, there is no action-at-a-distance in a sense that the correlations by themselves can not lead to information transfer between the points where the spins are located.

Current interest to the solid-state quantum information processing (see, e.g., the reviews [3, 4, 5]), motivates development of mesoscopic solid-state structures that can serve both as simple quantum systems, e.g., qubits or harmonic oscillators, and the detectors. Although in experiments, the mesoscopic detectors did not reach the stage yet where they can be used to look into the basic questions of the quantum measurement theory (which requires the quantum-limited detection) one can expect this to happen quite soon. A new element introduced by the mesoscopic structures in the discussion of quantum measurements is the fact that the wavefunction reduction is not necessarily caused by interaction of a “microscopic” measured system with the “macroscopic” detector. In mesoscopic structures, the measured systems and the detectors are similar in many respects (including dimensions and typical dynamics) and are frequently interchangeable: a measured system in one context can act as the detector in the other, and vice versa. This shows that the boundary at which the quantum coherent dynamics should be complemented with the wavefunction reduction is not universal.

The aim of this work is to provide a quantitative discussion of models and measurement dynamics of the mesoscopic detectors. The discussion emphasizes the interplay between the dynamic and information sides of the measurement process and can serve as an introduction to the problem of wavefunction reduction in mesoscopic structures.

2. – Measurements dynamics of ballistic mesoscopic detectors

Majority of the mesoscopic detectors use as their operating principle ability of a measured system to control the transport of some particles between the two reservoirs. The information about the state of the system is contained then in the magnitude of the particle current between the reservoirs which serves as the detector output. In the most direct form, this principle is implemented in the quantum point contact (QPC) detector [6, 7], which presently is the main detector used for measurements of the quantum dot qubits [8, 9, 10, 11, 12]. In the QPC detector (Fig. 1), the propagating particles are

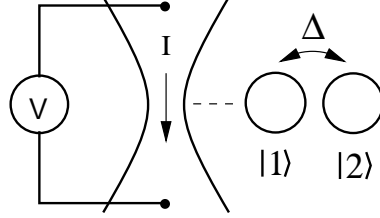


Fig. 1. – Schematic of the QPC detector measuring charge qubit. The two qubit states $|j\rangle$, $j = 1, 2$ are localized on the opposite sides of a tunnel barrier and are coherently coupled by tunneling across this barrier with coupling strength Δ . Transfer of the qubit charge between the states $|j\rangle$ changes electrostatic potential in the scattering region of the QPC affecting the current I through it that is driven by the applied voltage V .

electrons which move ballistically through a short one-dimensional constriction formed between the two electrodes of the QPC. The electrodes can be viewed as reservoirs of independent and effectively non-interacting electrons. The measured system creates electrostatic potential that makes the scattering potential $U_j(x)$ for electrons in the constriction dependent on the state $|j\rangle$ of the system, and in this way controls transmission probability of the QPC. The output of the QPC detector is the electric current I driven by the voltage difference V between the electrodes. The current depends on the electron transmission probability, and as a result, contains information about the state $|j\rangle$. Since interaction between the QPC electrons and the measured system is dominated by the electrostatic potential, the QPC acts as the charge detector. Another example of the ballistic mesoscopic detector is the magnetic analog of the QPC based on the ballistic motion of the magnetic flux quanta (fluxons) through a one-dimensional channel, the role of which is played by the Josephson transmission line (JTL) [13]. The scattering potential $U_j(x)$ for the fluxons in the JTL is created by the magnetic flux or current, and the JTL detector can be used for measurements of superconducting flux qubits. In the JTL detector, the fluxons can be injected into the JTL individually providing control over the individual scattering events.

The detector model in which the output information is contained in the transport current flowing between the two reservoirs applies to many of the mesoscopic detectors (see Sec. 4). There are several reasons for this. One is the strong (in the tunnel limit, exponential) dependence of the scattering amplitudes on parameters of the scattering potential that leads to sufficiently large sensitivity of the detector to the measured system. Another, more important, is the fact that the scattering dynamics contains strongly divergent transmitted and reflected trajectories that create easily detectable different outcomes of measurement. This feature of scattering is not easily reproducible in other types of the dynamics [14]. Finally, the transport between large reservoirs makes it possible to repeat scattering events at a certain rate amplifying the results of scattering of one particle.

In general, the process of quantum measurement can be understood as creation of

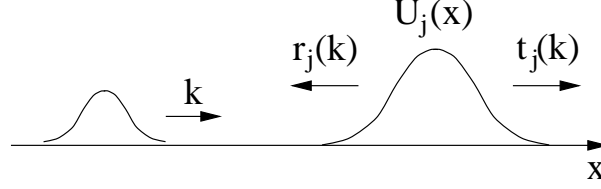


Fig. 2. – Measurement dynamics of a ballistic mesoscopic detector. The wavepacket of a particle with momentum k is scattered by the potential $U_j(x)$ controlled by the measured system. The scattering potential and the transmission/reflection amplitudes $t_j(k)$, $r_j(k)$ contain information about the state $|j\rangle$ of the system.

an entangled state of the measured system and the detector as a result of interaction between them. The states of the detector are classical and suppress quantum superposition of different outcomes of measurement. The two consequences of this process are the acquisition of information about the system by the detector, and “back-action” dephasing of the measured system - see, e.g., [15, 16]. Because of the system-detector entanglement, finding a given detector output provides some indication of what state the measured system is in. On the other hand, the same entanglement means that quantum coherence among the states of the measured system is suppressed. This implies that there is a close connection between the information acquisition and back-action dephasing. In the optimal situation, the rate W with which the detector obtains information about the system and the dephasing rate Γ are the same. Of course, the detector can always introduce some parasitic dephasing into the system dynamics, so that in general $W \leq \Gamma$. In view of this inequality, the detector with $W = \Gamma$ is called “ideal” or “quantum limited”. If the detector is far from being quantum-limited, it destroys quantum coherence in the measured system long before it provides information that can be used to select specific outcomes of measurement. Because of this, only the detectors that are close to being quantum-limited can give rise to non-trivial wavefunction reduction.

2'1. Back-action dephasing rate. – Measurement dynamics with the ballistic mesoscopic detector is illustrated in Fig. 2. For the ballistic detector, the detector-system entanglement arises as a result of scattering (Fig. 2), and the rates of information acquisition and back-action dephasing can be expressed in terms of the scattering amplitudes [17]. To do this, we consider evolution of the density matrix ρ of the measured system in scattering of one particle. For simplicity, the Hamiltonian of the system itself is assumed to be zero (e.g., $\Delta = 0$ in the example of Fig. 1), and the system evolution is caused only by the interaction with the detector. The evolution of ρ is obtained then from the time dependence of the total wavefunction of the scattered particle and the stationary wavefunction $\sum_j c_j |j\rangle$ of the measured system:

$$(1) \quad \psi(x, t=0) \cdot \sum_j c_j |j\rangle \rightarrow \sum_j c_j \psi_j(x, t) \cdot |j\rangle.$$

Here $\psi(x, t = 0)$ is the initial wavefunction of the particle injected in the scattering region from the reservoir, and its time evolution $\psi_j(x, t)$ depends on the realization $U_j(x)$ of the potential created by the measured system. Tracing over the detector, i.e., the scattering wavefunction, one gets from Eq. (1):

$$(2) \quad \rho_{ij} = c_i c_j^* \rightarrow c_i c_j^* \int dx \psi_i(x, t) \psi_j^*(x, t).$$

Qualitatively, the time evolution in (1) describes propagation of the initial wavepacket towards the scattering potential and its subsequent separation in coordinate space into the transmitted and reflected parts that are well-localized on the opposite sides of the scattering region. At time $t > t_{sc}$, where t_{sc} is the characteristic scattering time, the separated wavepackets move in the region free from the j -dependent potential and the unitarity of the quantum-mechanical evolution of $\psi_j(x, t)$ implies that the overlap of the scattered wavefunctions in Eq. (2) becomes independent of t . This overlap can be directly found in the momentum representation:

$$(3) \quad \int dx \psi_i(x, t) \psi_j^*(x, t) = \int dk |b(k)|^2 [t_i(k) t_j^*(k) + r_i(k) r_j^*(k)],$$

where $b(k)$ is the probability amplitude for the injected particle to have momentum k in the initial state. Equations (2) and (3) show that the diagonal elements of the density matrix ρ do not change in the scattering process:

$$(4) \quad \int dk |b(k)|^2 [|t_i(k)|^2 + |r_i(k)|^2] = 1,$$

while the off-diagonal elements are suppressed by the factor

$$(5) \quad \left| \int dk |b(k)|^2 [t_i(k) t_j^*(k) + r_i(k) r_j^*(k)] \right| \leq 1.$$

The inequality in this relation follows from the Swartz inequality for the scalar product in the Hilbert space of “vectors” $|b(k)| \cdot \{t_j(k), r_j(k)\}$ of the unit length (4). Suppression of the off-diagonal elements of ρ is manifestation of the back-action dephasing of the measured system by the detector. Assuming that the particles are injected from the reservoir with frequency f and combining the suppression factors (5) for the successive scattering events, we obtain the dephasing rate as

$$\Gamma_{ij} = -f \ln \left| \int dk |b(k)|^2 [t_i(k) t_j^*(k) + r_i(k) r_j^*(k)] \right|.$$

If the scattering amplitudes do not depend on momentum k in the range of momenta limited by $|b(k)|^2$, the back-action dephasing rate becomes independent of the form of

initial wavepacket of injected particles:

$$(6) \quad \Gamma_{ij} = -f \ln |t_i t_j^* + r_i r_j^*|.$$

Equation (6) is the general expression for the back-action dephasing rate of a ballistic mesoscopic detector with momentum- (and energy-) independent scattering amplitudes. It is valid, in particular, for the QPC detector at low temperatures $T \ll eV$, if the injection frequency f is taken to be equal to the “attempt frequency” eV/h with which electrons are incident on the scattering region [24].

In the linear-response regime, when the changes in the scattering amplitudes with j are small, the limiting form of Eq. (6) was obtained in [18, 19, 20, 21, 22]. In the tunnel limit $t_j \rightarrow 0$, Eq. (6) reduces to

$$(7) \quad \Gamma_{ij} = (f/2) |\bar{t}_i - \bar{t}_j|^2,$$

where

$$(8) \quad \bar{t}_j \equiv |t_j| e^{i\phi_j}, \quad \phi_j \equiv \arg(t_j/r_j).$$

When the phases ϕ_j can be neglected, Eq. (7) reproduces earlier results [23] for the dephasing by the QPC detector in the tunnel limit. As we will see in Sec. 4, Eq. (7) can be obtained in this limit under more general assumptions, and describes large number of different mesoscopic detectors.

2.2. Information acquisition rate. – As was discussed above, the back-action dephasing is only one part of the measurement process. The other part is acquisition by the detector of information about the state of the measured system. In the case of ballistic detector, the information is contained in the scattering amplitudes of the incident particles, and the rate of its acquisition depends on specific characteristics of the amplitudes recorded by the detector. One of the simplest possibilities in this respect, realized, e.g., in the QPC detectors, is to record the changes in the transmission probabilities which determine the magnitude of the particle current through the scattering region. (Alternatively, one could modify the scattering scheme by forcing the scattered particles to interfere, and in this way use the phase information [17] in the scattering amplitudes.) The rate of information extraction from the current magnitude, i.e., the rate of increase of the confidence level in distinguishing different states $|j\rangle$, can be calculated simply by starting with the transmission/reflection probabilities $T_j = |t_j|^2$ and $R_j = 1 - T_j$ when the measured system is in the state $|j\rangle$. Since successive scattering event are independent, the probability $p_j(n)$ to have n out of N incident particles transmitted, is given by the binomial distribution $p_j(n) = C_N^n T_j^n R_j^{N-n}$. The task of distinguishing different states $|j\rangle$ of the measured system is transformed by the detector into distinguishing the probability distributions $p_j(n)$ for different j s. Since the number $N = ft$ of scattering attempts increases with time t , the distributions $p_j(n)$ become peaked successively more strongly around the corresponding average numbers $T_j N$ of transmitted particles. The states

with different probabilities T_j can be distinguished then with increasing certainty. The rate of increase of this certainty can be characterized quantitatively by some measure of the overlap of the distributions $p_j(n)$. While in general there are different ways to characterize the overlap of different probability distributions [25], the characteristic which is appropriate in the quantum measurement context [26, 16] is closely related to “fidelity” in quantum information [25]: $\sum_n [p_i(n)p_j(n)]^{1/2}$. The rate of information acquisition can then be defined naturally as [17]:

$$(9) \quad W_{ij} = -(1/t) \ln \sum_n [p_i(n)p_j(n)]^{1/2}.$$

Using the binomial distribution in this expression we get:

$$(10) \quad W_{ij} = -f \ln[(T_i T_j)^{1/2} + (R_i R_j)^{1/2}].$$

Equation (10) gives the information acquisition rate by ballistic mesoscopic detectors. Comparing Eqs. (6) and (10) we see that for this type of the detector, in accordance with the general understanding of quantum measurements, the back-action dephasing rate and information acquisition rate satisfy the inequality

$$(11) \quad W_{ij} \leq \Gamma_{ij}.$$

Equality in this relation gives the condition of the quantum-limited operation of the ballistic mesoscopic detector under the assumption of energy-independent scattering amplitudes. It holds if

$$(12) \quad \phi_j = \phi_i,$$

where the phases ϕ_j are defined in Eq. (8). Condition (12) has simple interpretation as the statement that there is no information on the states $|j\rangle$ in the phases of the scattering amplitudes. Deviations from Eq. (12) mean that the phases contain information about the measured system which is lost in the detection scheme sensitive only to the transmission probabilities T_j . In this case, the information loss in the detector prevents it from being quantum-limited. In practical terms, the simplest way to satisfy condition (12) is to make the scattering potential symmetric $U_j(-x) = U_j(x)$ for all states $|j\rangle$. The unitarity of the scattering matrix for this potential implies then that $\phi_j = \pi/2$ for any j , and Eq. (12) is automatically satisfied. If the detector is quantum-limited, it can demonstrate non-trivial wavefunction reduction.

2.3. Conditional evolution. – Quantitative description of the wavefunction reduction due to interaction with a detector can be formulated as “conditional” evolution, in which dynamics of the measured system is conditioned on the observation of particular outcome of measurement. In the axiomatic approach, wavefunction reduction is formalized together with the dynamic evolution as “quantum operation” [27], arbitrary linear transformation of the system density matrix satisfying physically motivated axioms. In this

approach, a detector is characterized by a set of positive operators which correspond to all possible outcomes of measurements with this detector, or “positive operator valued measure” (POVM) [28]. In practice, for any real specific detector, it is clear what the possible classical outcomes of measurements are, and the emphasis is then on development of dynamic equations that would describe evolution of the measured system conditioned on a given detector output. Since the different outcomes of the detector evolution are classically distinguishable, it is meaningful to ask how the measured system evolves for a given output. Such a conditional evolution of the measured system describes quantitatively the wavefunction reduction in the measurement process with a particular detector (see, e.g., [29, 30, 31, 16]). In the case of ballistic mesoscopic detectors, each act of particle scattering represents an elementary measurement process. Since the particle trajectories that correspond to different outcomes of scattering: transmission through or reflection from the scattering region are strongly separated, these outcomes should be considered as non-interfering classical events. Although the absence of quantum coherence between these two outcomes is an assumption of the conditional approach, this assumption is very natural. Propagation of the scattered particles in different reservoirs of the detector entangles them with different environments, the process that very efficiently suppresses their mutual quantum coherence [33]. While this “common-sense” assumption of absence of quantum coherence between different outputs of a realistic detector is sometimes considered unsatisfactory from an abstract point of view [34, 35], it can be given a fairly rigorous description in terms of decoherence in open quantum systems – see, e.g., [36].

Quantitatively, conditional equations are obtained by separating in the total wavefunction the terms that correspond to a specific classical outcome of measurement and renormalizing this part of the wavefunction so that it corresponds to the total probability of 1 [37, 16, 13]. In the ballistic detector, there are two classically different outcomes of scattering, transmission and reflection, for each injected particle. This means that the wavefunction of the measured system should be conditioned on the observation of either transmitted or reflected particle in each elementary cycle of measurement. The evolution of the total wavefunction “detector+measured system” as a result of scattering of one particle is described by Eq. (1). Under the assumption of energy-independent scattering amplitudes, momentum and coordinate dependence of the states of the scattered particles in the detector is the same for different states $|j\rangle$, and can be factored out from the total wavefunction. The evolution of the measured system can then be conditioned on the transmission/reflection of a particle simply by keeping in Eq. (1) the terms that correspond to the actual outcome of scattering in the form of the appropriate scattering amplitudes. If the particle is transmitted through the scattered region or reflected from it in a given measurement cycle, amplitudes c_j for the system to be in the state $|j\rangle$ change then, respectively, as follows:

$$(13) \quad c_j \rightarrow t_j c_j / [\sum_j |c_j|^2 T_j]^{1/2}, \quad c_j \rightarrow r_j c_j / [\sum_j |c_j|^2 R_j]^{1/2}.$$

We see that the expansion coefficients c_j of the system’s wavefunction are changing in conditional evolution despite the initial assumption that the system Hamiltonian is

zero. This is unusual from the point of view of the Schrödinger equation, and provides quantitative expression of reduction of the wavefunction in the measurement process.

Also, it should be noted that the transformations (13) do not decohere the measured system despite the back-action dephasing by the detector discussed above. To understand this, one should note that as in the case of any dephasing, the back-action dephasing can be viewed as the loss of information. For the quantum-limited detection, the overall evolution of the detector and the measured system is quantum-coherent and the only possible source of the information loss is averaging over the detector evolution. This means that the back-action dephasing arises as the result of averaging over different measurement outcomes [32], and specifying definite outcome as done in the conditional dynamics removes all losses of information and eliminates the dephasing.

Equations (13) can be applied directly to the detectors which provide control over scattering of individual particles, e.g. to the JTL detector [13], where it makes sense to discuss changes in the wavefunction of the measured system induced by one scattering event. In some detectors, however, such a control over individual scattering events is not fully possible. For instance, in the QPC detector, the picture of individual scattering events leading to Eqs. (6) and (10) for the back-action dephasing and information rates is strictly speaking valid only on the relatively large time scale $t \gg h/eV$, when the typical number of electron scattering attempts is larger than 1. One can generalize Eq. (13) to this situation by considering the time interval t which includes a number $N = ft > 1$ of scattering attempts, where for the QPC $f = eV/h$. Combining the transformations (13) one can see that observation of any sequence of transmission/reflection events that includes n transmissions and $N - n$ reflections changes the wavefunction as:

$$(14) \quad c_j \rightarrow t_j^n r_j^{(N-n)} c_j / \left[\sum_j |c_j|^2 T_j^n R_j^{(N-n)} \right]^{1/2},$$

regardless of the specific ordering of these events. This equation includes as a particular case Eq. (13) which follows when $N = 1$. Since the wavefunction obtained as a result of transformation (14) is the same for all C_N^n sequences with the same total number n of transmitted particles, one can distinguish all scattering outcomes only by n . For each of the $N + 1$ outcomes with different n the wavefunction is transformed according to Eq. (14). This means that the wavefunction reduction has the form (14) independently of whether the detector suppresses quantum coherence between all the sequences of scattering outcomes or only between the states with different total numbers of transmissions/reflections. Transformations (14) can be used to study quantitatively unusual manifestations of the wavefunction reduction in the mesoscopic solid-state qubits.

3. – Tunneling without tunneling: wavefunction reduction in a mesoscopic qubit

Probably the simplest example of the counter-intuitive features of the wavefunction reduction in mesoscopic qubits arises from a question whether a quantum particle can tunnel through a barrier which has vanishing transparency? Immediate answer to this

question is “no” as follows from the elementary properties of the Schrödinger equation. It seems basically the tautology to say that if the tunneling amplitude is zero (e.g., the barrier is infinitely high) the tunneling is suppressed. Of course somewhat more careful consideration reminds that evolution according to the Schrödinger equation is not the only way for a state of a quantum particle to change in time. Changes in the particle state can also be caused by the wavefunction reduction, which, as discussed also in the Introduction, can in principle violate any feature of dynamic evolution of a quantum system. This can be expressed quantitatively through the “Bell” inequalities generalizing the classic Bell’s inequalities which quantify violation of the no-action-at-a-distance principle. For measurements of a mesoscopic qubit (Fig. 1), the peculiarities of quantum dynamics of the system originate from the possibility of quantum-coherent uncertainty in the position of the charge or flux between the two basis states $|j\rangle$ of the qubit. In the regime of coherent oscillations of the qubit ($\Delta \neq 0$), this uncertainty gives rise to several “temporal” Bell inequalities [38, 39, 40, 41]. In this Section, we discuss a sequence of quantum transformation that is centered around the qubit dynamics with suppressed tunneling, $\Delta = 0$. The transformations lead to the associated Bell-type inequality, which quantify violation of the fundamental intuition of many solid-state physicists: charge or magnetic flux can not tunnel through an infinitely large barrier. The sequence of transformation includes a measurement done on the qubit and shows that the wavefunction reduction can indeed violate this Schrödinger-equation-based intuition, and a particle can be transferred through an “impenetrable” barrier in the process of quantum measurement.

The required manipulations of the qubit state are close to those in current experiments on coherent oscillations and more complex dynamics of mesoscopic qubits. Although the discussion in this Section applies in general to all types of qubits, the physics content of the wavefunction reduction is more striking for the semiconductor [9, 11, 12] or superconductor [42, 43, 44, 45, 46] charge qubits, or for the flux qubits [47, 48, 49]. In this case, the basic set-up is equivalent to the one shown in Fig. 1. The two basis states $|j\rangle$, $j = 1, 2$, of the qubit differ by some amount of magnetic flux or by an individual charge (electron charge e in semiconductor quantum dot qubit, or Cooper-pair charge $2e$ in a superconductor qubit) localized on the opposite sides of a tunnel barrier. The states are coupled by the tunnel amplitude $\Delta > 0$. At the point of resonance, when the bias energy ϵ between the two basis states vanishes, the qubit Hamiltonian reduces to

$$(15) \quad H = -\Delta\sigma_x,$$

and describes quantum coherent oscillations with frequency 2Δ . The tunnel amplitude Δ and the bias energy ϵ are assumed to be controlled externally.

The main element of the sequence of qubit transformations consists in preparing a superposition (for simplicity, symmetric superposition, $\sigma_x = 1$) of the qubit basis states, then switching off the tunnel amplitude Δ and performing quantum-limited but weak measurement of the qubit position in the σ_z basis. The idea is to prove that the measurement-induced transfer of the qubit wavefunction between the two qubit states at

$\Delta = 0$ gives not only the changes of the probability (representing our knowledge about the qubit position) but the actual transfer of charge or flux through the infinitely large barrier. In order to do this, we include the measurement-induced wavefunction transfer as a part of the cyclic transformation, the other part of which is known to transfer charge or flux. In the ideal situation, when the operations are precise and there is no intrinsic decoherence, the cycle should lead to precisely the same initial state $\sigma_x = 1$, making it possible to conclude that the measurement part of it transferred the qubit state through the barrier with vanishing tunneling amplitude. In the presence of external perturbations, there will be a probability $p^{(-)}$ to find the qubit not in the initial state. One can, however, derive a condition in the form of inequality on $p^{(-)}$, violation of which shows that this observation cannot be explained within the assumption of some initial classical probability distribution over the qubit basis states. This means that explanation of the observed violation should necessarily involve the transfer of the qubit state through suppressed barrier by the process of the wavefunction reduction.

In detail, the starting point of the sequence of transformations is the ground state of the Hamiltonian (15)

$$(16) \quad |\psi_0\rangle = (|1\rangle + |2\rangle)/\sqrt{2},$$

in which the qubit will find itself because of the unavoidable, but assumed to be weak, relaxation processes, if the Hamiltonian (15) is kept stationary for some time. Starting from this state, the tunnel amplitude Δ is abruptly switched off, $\Delta = 0$. The rate of this process is not important in the case of the Hamiltonian (15), since the final state will coincide with (16) regardless of how slowly or quickly Δ is switched off. However, in the presence of some parasitic residual bias ϵ , the rate of variation of Δ should be much larger than ϵ to preserve the state (16) at the end of the switching process. Next, as the first step of actual transformations of the qubit state (16), the weak quantum-limited measurement of the σ_z operator is performed on this state. The result of this operation does not depend on the specific model of measurement, as long as it is quantum-limited. For such a measurement, we know that specifying the detector output n leaves the qubit in a pure state which is obtained from (16) by increased degree of localization in the σ_z basis because of the information about σ_z provided by the measurement:

$$(17) \quad |\psi_1\rangle = \alpha_n|1\rangle + \beta_n|2\rangle.$$

As a suitable model of this measurement one can use the measurement with a ballistic detector discussed above. In this case, the detector output is the number n of transmitted particles in $N = ft$ scattering attempts. In this case, according to Eq. (14)

$$\alpha_n = \frac{t_1^n r_1^{(N-n)}}{\left[T_1^n R_1^{(N-n)} + T_2^n R_2^{(N-n)}\right]^{1/2}}, \quad \beta_n = \frac{t_2^n r_2^{(N-n)}}{\left[T_1^n R_1^{(N-n)} + T_2^n R_2^{(N-n)}\right]^{1/2}}.$$

Using the condition (12) of the detector ideality one can see that the relative phase ξ of the

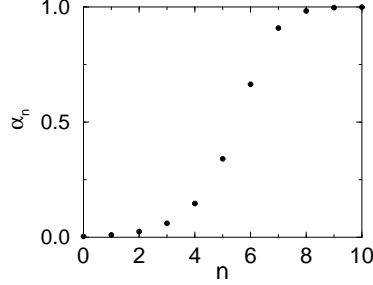


Fig. 3. – Probability amplitude α_n (18) of finding the qubit in the state $|1\rangle$ as a function of the observed number n of particles transmitted in $N = 10$ attempts through the ballistic detector with transmission probabilities $T_1 = 0.8$ and $T_2 = 0.4$. The detector measures the state (16).

coefficients α_n and β_n is independent of the detector output n : $\xi = N[\arg(t_1) - \arg(t_2)]$. It can be viewed then as renormalization of the qubit bias ϵ due to the detector-qubit coupling, and can be compensated for by the bias shift $\delta\epsilon = f[\arg(t_1) - \arg(t_2)]$ during the period of measurement. The coefficients α, β have then the following form:

$$(18) \quad \alpha_n = \left[\frac{w_1^{(n)}}{w_1^{(n)} + w_2^{(n)}} \right]^{1/2}, \quad \beta_n = \left[\frac{w_2^{(n)}}{w_1^{(n)} + w_2^{(n)}} \right]^{1/2},$$

where

$$w_j^{(n)} = T_j^n R_j^{(N-n)}$$

can be interpreted as the relative probability for the qubit to be in the state $|j\rangle$ for a given detector outcome n . In the situation when the detector provides no information on σ_z , e.g. if $T_1 = T_2$, the coefficients (18) are unchanged from their initial values (16). Otherwise, the probability amplitude is shifted in the direction of the more probable state: the amplitude (18) of one qubit state is increased in comparison with (16) if the observed n is closer to the value $n_j = T_j N$ characteristic for this state than the other state. As an illustration, Fig. 3 shows the amplitude α_n for $N = 10$, $T_1 = 0.8$, and $T_2 = 0.4$, as a function of n . One can see that α_n maintains its original value $1/\sqrt{2}$ from Eq. (16) for n 's roughly in the middle between the two characteristic values $n_1 = 8$ and $n_2 = 4$. For n smaller than or close to n_2 , α_n decreases from its original value, for n close to or larger than n_1 , α_n approaches 1. Such a shift due to the wavefunction reduction is the central part of the transformation cycle.

The remaining steps of the cycle aim at returning the qubit to its initial state (16). To do this, one needs to transfer back the charge or flux that was transferred in the wavefunction reduction process leading to the state (18). This is achieved by creating for some time the non-vanishing tunneling amplitude, i.e. realizing a fraction of a period of the regular coherent oscillations in which the charge or flux goes back-and-forth between

the two qubit basis states. In the most direct way, this can be done if the qubit structure makes it possible to create non-vanishing phase of the tunnel amplitude $\Delta'(t)$ (e.g., in the superconducting qubits, where the tunnel amplitude can be controlled through quantum interference, producing any complex value of this amplitude). In this case, the state (18) can be returned back directly into the initial form (16) if $\arg \Delta' = \pi/2$. In the diagram (Fig. 4a) in which the qubit states are represented in the language of spin-1/2, i.e.

$$|\psi_1\rangle = \cos(\theta_n/2)|1\rangle + \sin(\theta_n/2)|2\rangle,$$

such a tunneling amplitude corresponds to rotation around the y axis. The diagram in Fig. 4a shows then directly that the rotation around y axis turning $|\psi_1\rangle$ into $|\psi_0\rangle$ should have the magnitude:

$$(19) \quad \int |\Delta'(t)| dt/\hbar = (\pi/2 - \theta_n)/2,$$

where

$$\theta_n = 2 \tan^{-1}(\beta_n/\alpha_n) = 2 \tan^{-1} \left[(T_2/T_1)^{n/2} (R_2/R_1)^{(N-n)/2} \right].$$

If the qubit structure allows only for the real tunnel amplitude Δ (the situation that can be expected, e.g., in semiconductor quantum dot qubits), the y -axis rotation $R_y = \exp\{-i\sigma_y \int |\Delta'(t)| dt/\hbar\}$ (19) can be simulated in three steps in which the rotation $R_x = \exp\{-i\sigma_x \int \Delta(t) dt/\hbar\}$ around the x axis of the same magnitude (19) is preceded and followed by the rotations around the z -axis:

$$(20) \quad R_y = U^{-1} R_x U, \quad U = \exp\{i\sigma_z \pi/4\}.$$

The z -axis rotations can be created by the pulses of the qubit bias: $\int \epsilon(t) dt/\hbar = \pm\pi/4$. The three-step sequence (20) can be simplified into two steps (Fig. 4b) by changing the order of rotations: first, the x -axis rotation by $\pi/4$ (opening tunneling $\Delta(t)$ for appropriate interval of time) followed by one z -axis rotation:

$$(21) \quad \int \Delta(t) dt/\hbar = \pi/4, \quad \int \epsilon(t) dt/\hbar = (\pi/2 - \theta_n)/2.$$

All these versions of the transformation cycle bring the qubit to its initial state (16). In all cases, completion of the cycle that started with a shift of the wavefunction amplitudes due to the state reduction involves a part of a period of coherent oscillations which reverses this shift. Coherent qubit oscillations are known to actually transfer the charge of flux between the two qubit states. Since the cycle as a whole is closed, this fact shows that the changes in the qubit state caused by the wavefunction reduction can not be interpreted only as the changes in our knowledge of probabilities of the state of the qubit, but involve the actual transfer of the charge or flux in the absence of the tunneling amplitude.

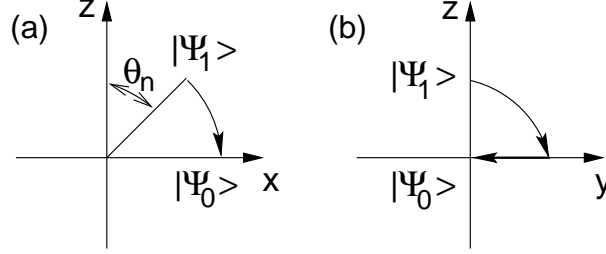


Fig. 4. – Diagram of the two possible transformations of the qubit state in the spin representation returning the state $|\psi_1\rangle$ to $|\psi_0\rangle$ after the measurement-induced state reduction $|\psi_0\rangle \rightarrow |\psi_1\rangle$. (a) Direct one-step y -axis rotation (19). (b) Projection on the $z - y$ plane of the two-step transformation (21) with the same end result.

To see this more quantitatively, one can derive the Bell-type inequality, violation of which should show that understanding of the state reduction solely in terms of probability changes can not be correct. The inequality is obtained by assuming that the process of switching off the tunneling amplitude Δ in the beginning of the transformation cycle does not leave the state (16) unchanged but instead localizes the qubit in one of the basis states. This means that the process produces an incoherent mixture of the qubit states with some, in general unspecified, probability p to be in the state $|1\rangle$. This process would provide then an alternative, classical description of the evolution during the measurement process. In this description, the qubit state is “objectively” well defined, but is unknown to us, and the measurement gradually provides information about this unknown state. The measurement would only change the probabilities we ascribe to the two qubit states, but not the state itself, and in particular would not transfer the charge of flux. One should then see how well this classical description can mimic the quantum result of the transformation cycle described above. A convenient way of making this comparison is provided by the probability of ending the cycle in the wrong state. The unperturbed quantum evolution should end up in the initial state $\sigma_x = 1$, whereas the same transformation cycle performed on the classical initial state will always have a finite probability $p^{(-)}$ of ending in the state $\sigma_x = -1$.

This probability is found by applying the transformations not to the state (16) but to the incoherent state with the density matrix

$$\rho_0 = p|1\rangle\langle 1| + (1-p)|2\rangle\langle 2|.$$

In this case, the measurement changes only the probability p in this expression. Similarly to Eq. (18), if the detector gives the output n , the density matrix of the system is

$$(22) \quad \rho_1 = \rho_0|_{p \rightarrow \bar{p}}, \quad \bar{p} = \frac{pw_1^{(n)}}{pw_1^{(n)} + (1-p)w_2^{(n)}}.$$

All versions (19) – (21) of the transformation cycle produce the same probability $p^{(-)}$ of being in the state $\sigma_x = -1$, when applied to the density matrix ρ_1 (22). For instance, one can see directly that in the density matrix $R_y^{-1}\rho_1 R_y$ obtained from ρ_1 by rotation (19), the probability $p^{(-)}$ is:

$$p^{(-)} = \frac{w_1 w_2}{(w_1 + w_2)(p w_1 + (1 - p) w_2)}.$$

Minimizing this expression with respect to p , we see that the minimum probability of finding the state $\sigma_x = -1$ in the classical case is

$$(23) \quad p^{(-)} = \frac{\min\{w_1, w_2\}}{w_1 + w_2}.$$

Instead of looking for the minimum with respect to p , one can adopt a natural additional assumption that when the tunneling amplitude is switched off, the qubit localization process can only be symmetric, since there is no reason to prefer one qubit state to another. In this case, $p = 1/2$, and we obtain somewhat different expression for the probability $p^{(-)}$ with qualitatively similar properties:

$$(24) \quad p^{(-)} = \frac{2w_1 w_2}{(w_1 + w_2)^2}.$$

This expression would also be obtained if the qubit wavefunction would be reduced to the density matrix ρ_1 (22) during the measurement, not in the process of suppression of the tunneling amplitude.

Equations (23) and (24) show that in order to distinguish the quantum coherent evolution (for which $p^{(-)} = 0$) and incoherent evolution with non-vanishing probability $p^{(-)}$, it is important to employ weak measurement. If the measurement is projective, i.e. if one of the probabilities w_j is zero so that the measurement completely reduces the qubit state to one of the basis states, then $p^{(-)} = 0$ and it is impossible to distinguish the two types of evolution. This conclusion should be independent of the specific form of the employed transformation cycle, since projective measurement is always expected to fully separate different components of the initial state of the measured system and completely suppress quantum coherence between them.

The discussion above means that observation of the probability of the state $\sigma_x = -1$ smaller than $p^{(-)}$,

$$(25) \quad p(\sigma_x = -1) < p^{(-)}$$

at the end of the transformation cycle proves that all transformations in this cycle, including the wavefunction reduction, are quantum coherent. Combined with the non-vanishing transfer of charge or flux during the “oscillation” step [(19) – (21)] of the cycle, this fact implies that the wavefunction reduction induces similar transfer across

the tunnel barrier separating the qubit basis states even if the corresponding tunnel amplitude is zero.

It is important to note that this counter-intuitive feature of the wavefunction reduction does not contradict the fact that all dynamic properties of the measurement, including the back-action dephasing and information rates can be calculated from the dynamic detector model without any reference to the wavefunction reduction. While the dynamic properties are average characteristics of the detector, the wavefunction reduction appears if one considers separately individual outcomes of measurement. In the transformations discussed above, this separation is achieved by introducing the feedback, operations on the measured system which depend on the specific measurement outcome. In ballistic detectors, the measurement outcomes are distinguished by the number n of transmitted particles, and to see the wavefunction reduction one needs to distinguish individual particles, and is done, e.g., in the electron counting experiments [50]. For the detectors for which distinguishing individual transmitted particles can be problematic (e.g., the QPC detector), the allowed uncertainty in n should be smaller than the width δn of the transition region in the n -dependence of the wavefunction amplitudes of the measured qubit – see Fig. 3. Since the width of this region can be estimated roughly as $\delta n \simeq 1/\ln(T_1 R_2/T_2 R_1)$, the uncertainty in n can be compensated for by making the difference between transmission probabilities T_j smaller, thus increasing δn . The limit to this increase is set by the decoherence processes in the measured system which make it impossible to increase the measurement time of the detector beyond the coherence time of the system without losing the non-trivial character of the measurement dynamics.

4. – Tunneling detectors

So far, the discussion was based on the ballistic model of the mesoscopic detector, in which the measured system controls ballistic motion of some particles between the two reservoirs (Fig. 2). If one assumes that the particle transmission probabilities are small, $T_j \ll 1$, and the transfer processes between the reservoirs can be described in the tunneling approximation, specific nature of the detector transport becomes irrelevant. In this case, the range of applicability of the detector model can be extended significantly to include the detectors in which it is not possible to identify regions of ballistic transport, but which are still based on the very similar dynamic principle: control by the measured system of transport between the two reservoirs. Examples of such “tunneling” detectors include the superconducting SET electrometer [51, 52, 20], normal SET electrometer in the co-tunneling regime [53], or dc SQUID magnetometer (see, e.g., [54, 55, 56, 57]) used for measurements of superconducting qubits. The aim of this Section is to show briefly that the measurement properties of this type of mesoscopic detectors coincide in essence with those of the ballistic detectors.

Since the measured system controls the tunneling amplitude \hat{t} of particles in the detector, this amplitude should be treated as a non-trivial operator acting on the measured system. The detector tunneling can be described then with the standard tunnel Hamiltonian, the transfer terms in which are split into a product of operators of the

measured system and the detector. In this case, the tunnel Hamiltonian describes the detector-system coupling and can be written as

$$(26) \quad H_T = \hat{t}\xi + \hat{t}^\dagger\xi^\dagger,$$

where ξ, ξ^\dagger are the operators that describe the detector part of the tunneling dynamics, e.g., creation of excitations in the detector reservoirs when a particle tunnels, respectively, forward and backward between them. Inclusion of the operators ξ, ξ^\dagger means, therefore, that the Hamiltonian (26) makes it possible to describe the detectors in which the tunneling processes are strongly inelastic. This fact, however, does not prevent correct account of elastic transport in the case of ballistic detectors. Qualitative reason for this can be seen easily using as an example the QPC detector. While scattering of individual electrons in the QPC is elastic, in the tunnel limit, electron transfer between the two electrodes of the QPC can also be viewed as creation of electron-hole excitation in the electrodes, with an electron removed from a state below the Fermi level in one electrode, and transferred to a state above the Fermi level in the other. Accordingly, as we will see later, Hamiltonian (26) leads to the evolution equations that coincide in the tunnel limit with those obtained above in the ballistic case.

Under the assumption that the detector tunneling is weak, the precise form of the internal detector Hamiltonian is not important and dynamics of measurement is defined by the correlators of the operators ξ, ξ^\dagger :

$$(27) \quad \gamma_+ = \int_0^\infty dt \langle \xi(t)\xi^\dagger \rangle, \quad \gamma_- = \int_0^\infty dt \langle \xi^\dagger(t)\xi \rangle.$$

Here the angled brackets denote averaging over the detector reservoirs which are taken to be in a stationary state with some fixed number of particles in them and the density matrix ρ_D : $\langle \dots \rangle = \text{Tr}_D \{ \dots \rho_D \}$. The correlators (27) set the scale $\Gamma_\pm \equiv 2\text{Re}\gamma_\pm$ of the forward and backward tunneling rates in the detector.

A reasonable tunneling detector should satisfy some additional assumptions related to the fact that its output should be classical in order to provide a complete measurement dynamics. Similarly to the ballistic detector, the output information in the tunneling case is contained in the number n of the particles transmitted between the detector reservoirs. For this number to behave classically, the correlators $\langle \xi(t)\xi \rangle, \langle \xi^\dagger(t)\xi^\dagger \rangle$ that do not conserve the number of tunneling particles should vanish. Another consequence of the assumption of classical detector output is that the energy bias ΔE for tunneling through the detector should be much larger than the typical energies of the measured system. In this case, one can neglect quantum fluctuations of the detector current in the relevant frequency range that corresponds to frequencies of evolution of the measured system. If one assumes in addition that all other characteristic frequencies of the detector tunneling are also much larger than those of the measured system, the functions $\xi(t), \xi^\dagger(t)$ in Eq. (27) are effectively δ -correlated on the time scale of the system.

Vanishing correlation time in the correlators (27) makes it possible to write down simple evolution equations for the density matrix ρ of the measured system. To describe

the system dynamics conditioned on particular outcome of measurement, we also keep in the evolution equation the number n of particles transferred through the detector. Since the correlators that do not conserve n vanish, only the terms diagonal in n are important. In the interaction representation with respect to the tunnel Hamiltonian (26), the density matrix $\rho(t)$ is given by the standard expression:

$$(28) \quad \rho(t) = \langle S \rho_D S^\dagger \rangle, \quad S = T \exp\{-i \int^t dt' H_T(t')\}.$$

For δ -correlated operators in Eq. (27), one can see that the full perturbation expansion of Eq. (28) in H_T is equivalent to the evolution equation for $\rho(t)$ that follows from the lowest-order perturbation theory. Keeping track of the number n of particles transferred through the detector, we get:

$$(29) \quad \dot{\rho}^{(n)} = \Gamma_+ \hat{t}^\dagger \rho^{(n-1)} \hat{t} + \Gamma_- \hat{t} \rho^{(n+1)} \hat{t}^\dagger - (\gamma_+ \hat{t} \hat{t}^\dagger + \gamma_- \hat{t}^\dagger \hat{t}) \rho^{(n)} - \rho^{(n)} (\gamma_+^* \hat{t} \hat{t}^\dagger + \gamma_-^* \hat{t}^\dagger \hat{t}).$$

Since the tunneling amplitude \hat{t} is a function of some observable of the measured system, there is a system of eigenstates $|j\rangle$ common to the operators \hat{t} and \hat{t}^\dagger :

$$\hat{t}|j\rangle = t_j|j\rangle, \quad \hat{t}^\dagger|j\rangle = t_j^*|j\rangle,$$

where t_j is the detector tunneling amplitude when the measured system is in the state $|j\rangle$. It is convenient to write the evolution equation (29) in the basis of states $|j\rangle$:

$$(30) \quad \begin{aligned} \dot{\rho}_{ij}^{(n)} = & -(1/2)(\Gamma_+ + \Gamma_-)(|t_i|^2 + |t_j|^2)\rho_{ij}^{(n)} + \Gamma_- t_i t_j^* \rho_{ij}^{(n+1)} \\ & + \Gamma_+ t_i^* t_j \rho_{ij}^{(n-1)} - i[\delta H, \rho]_{ij}, \end{aligned}$$

where

$$(31) \quad \delta H = \text{Im}(\gamma_+ + \gamma_-)|t_j|^2|j\rangle\langle j|$$

is the renormalization of the Hamiltonian of the measured system due to its coupling to the detector.

If one omits the term δH which can be combined with the internal Hamiltonian of the measured system, Eq. (30) can be solved in n by noticing that it coincides in essence with a recurrence relations for the modified Bessel functions I_n [58]. In order to interpret n as the number of particles transferred through the detector during the time interval t , we solve this equation with the initial condition $\rho_{ij}^{(n)}(t=0) = \rho_{ij}(0)\delta_{n,0}$. The corresponding solution is:

$$(32) \quad \frac{\rho_{ij}^{(n)}(t)}{\rho_{ij}(0)} = \left(\frac{\Gamma_+}{\Gamma_-}\right)^{n/2} I_n(2t|t_i t_j| \sqrt{\Gamma_+ \Gamma_-}) \exp\left\{-\frac{\Gamma_+ + \Gamma_-}{2}(|t_i|^2 + |t_j|^2)t - in\varphi_{ij}\right\},$$

where $\varphi_{ij} \equiv \arg(t_i t_j^*)$. As discussed above, the qubit density matrix conditioned on the particular measurement outcome n follows from Eq. (32) if one selects in this equation the terms with given n and normalizes the resulting reduced density matrix back to 1. If one of the tunneling rates Γ_+ or Γ_- vanishes, Eq. (32) reduces to the usual Poisson distribution characteristic for tunneling in one direction. Conditional description of measurement in this case was developed in [16]. When both rates are non-vanishing, specifying the total number n of the transferred particles does not specify uniquely evolution of the detector, since the same n results from the balance between different numbers of particles transferred forward and backward. This means that some information is lost in this regime and the detector is not quantum-limited (see the discussion below). In contrast to the situation with the quantum-limited detectors considered in the previous Sections, conditional dynamics that follows from Eq. (32) in this case [59] does not preserve the purity of the quantum state of the measured system.

Evolution of the density matrix ρ averaged over the measurement outcomes n can be obtained by either disregarding simply the index n in Eq. (30), or directly taking the sum over n in (32) with the help of a summation formula [58] for the Bessel functions. In both cases, equation for the measurement-induced evolution of ρ is:

$$(33) \quad \dot{\rho}_{ij} = -\Gamma_{ij}\rho_{ij} - i(\Gamma_+ - \Gamma_-)|t_i t_j| \sin \varphi_{ij} \rho_{ij},$$

where

$$(34) \quad \Gamma_{ij} \equiv (1/2)(\Gamma_+ + \Gamma_-)|t_i - t_j|^2$$

is the back-action dephasing rate by the tunneling detector. The last term in Eq. (33) can be viewed as another contribution to renormalization of the energy difference between the states $|i\rangle$ and $|j\rangle$, although in general it can not be reduced to the energy shifts of individual states [in contrast to the renormalization term (31)].

The back-action dephasing rate (34) coincides with that of the ballistic detectors in the tunneling limit given by Eq. (7), if we take into account that our discussion of ballistic detectors assumed for simplicity that particles are incident on the scattering region from only one electrode. It can be seen directly that the difference in the phases [see Eq. (8)] of the tunneling amplitudes in the two expressions is not essential. The reason for this difference is that the tunnel Hamiltonian (26) describes explicitly only the effects associated with the actual transfer of particles across the detector. It is assumed that the other effects of the detector-system coupling, e.g., renormalization of the system energy due to reflection processes in the detector, are already accounted for. In terms of the scattering amplitudes, this implies that the reflection amplitudes r and \bar{r} for particles incident on the tunnel barrier from the two electrodes of the detector, should satisfy the conditions $\arg(r_i r_j^*) = 0$ and $\arg(\bar{r}_i \bar{r}_j^*) = 0$. In this case, Eq. (7) coincides with (34) even with account of extra phases (8). This means that the model of the tunnel detector considered in this Section is equivalent to that of the ballistic detector in the appropriate small-transparency limit.

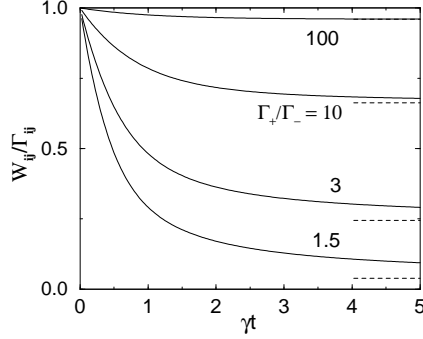


Fig. 5. – The information acquisition rate W_{ij} of the tunneling detector normalized to the dephasing rate Γ_{ij} (34) (for $\varphi_{ij} = 0$) as a function of time t for several ratios of the forward and backward tunneling rates. The time t is normalized to the typical forward tunneling rate $\gamma = \Gamma_+ (T_i + T_j)/2$. The curves are plotted for the detector transparencies $T_i = 0.2$, $T_j = 0.4$. The dashed lines show the corresponding asymptotic values (35). For $\Gamma_+/\Gamma_- = 100$, the dashed line overlaps with the main curve.

To calculate the information rate W_{ij} of the tunneling detector in the situation when the rates of both forward and backward tunneling are non-vanishing (in contrast to scattering from one direction discussed for the ballistic detectors), one needs to use in Eq. (9) the diagonal part of Eq. (32) which gives the probabilities $p_j(n) = \rho_{jj}^{(n)}$ for n particles to tunnel when the system is in the state $|j\rangle$. An example of the rate W_{ij} defined in this way is shown in Fig. 5. This figure shows that in general W_{ij} is time-dependent and approaches constant value only after a transition period. If the tunneling probabilities $T_j = |t_j|^2$ do not differ very strongly, this transition period is shorter than the back-action dephasing time Γ_{ij}^{-1} .

The constant asymptotic values of the information rate can be obtained from the asymptotic behavior of the Bessel functions $I_n(z)$. Using the standard integral representation for $I_n(z)$ in Eqs. (9) and (32), and making use of the fact that the constant rates W_{ij} are determined by the exponential behavior of the integrals at large time t , we find directly

$$(35) \quad W_{ij} = (\Gamma_+ + \Gamma_-)(T_i + T_j)/2 - [(T_i^2 + T_j^2)\Gamma_+\Gamma_- + T_iT_j(\Gamma_+^2 + \Gamma_-^2)]^{1/2}.$$

If the particles tunnel only in one direction, e.g. $\Gamma_- = 0$, Eq. (35) reduces to the previously known result [23, 32] $W_{ij} = \Gamma_+(\sqrt{T_i} - \sqrt{T_j})^2/2$, in which the information rate and the back-action dephasing rates coincide, when the phases of the tunneling amplitudes satisfy the appropriate ideality condition $\varphi_{ij} = 0$. In the other limit of small difference $2\Delta T$ between the transmission probabilities $T_{i,j} = T \pm \Delta T$, Eq. (35) reduces

to

$$(36) \quad W_{ij} = \frac{(\Delta T)^2 (\Gamma_+ - \Gamma_-)^2}{2T(\Gamma_+ + \Gamma_-)}.$$

This equation agrees with the general theory of linear measurements (see, e.g., [15]), in which it can be interpreted as the rate with which one can distinguish the difference $\Delta T(\Gamma_+ - \Gamma_-)$ between the two detector currents in the presence of the current noise with spectral density $T(\Gamma_+ + \Gamma_-)$.

In the most typical situation, the two rates Γ_{\pm} are both non-vanishing because of the finite temperature Θ of the detector electrodes. The electrodes can be in equilibrium even when a non-vanishing current is driven between them by finite energy difference ΔE created for the tunneling particles. In this case, the tunneling rates Γ_{\pm} are related by the detailed balance relation and can be written as $\Gamma_{\pm} = \Gamma_0 \exp(\pm \Delta E / 2\Theta)$, where Γ_0 is the typical tunneling rate which can also depend on temperature and energy bias. The information rate (35) then is

$$(37) \quad W_{ij}/\Gamma_0 = (T_i + T_j) \cosh(\Delta E / 2\Theta) - [T_i^2 + T_j^2 + 2T_i T_j \cosh(\Delta E / \Theta)]^{1/2}.$$

Comparison of Eqs. (37) or (35) with Eq. (34) for the back-action dephasing rate (see also Fig. 5) shows that the detector with temperature $\Theta \sim \Delta E$ which creates non-vanishing rates Γ_{\pm} , is not quantum-limited, $W_{ij} < \Gamma_{ij}$, even if the phases of the tunneling amplitudes satisfy the ideality condition $\varphi_{ij} = 0$. Equation (37) can be used to establish quantitative condition on the detector temperature necessary for the desired degree of the detector ideality.

5. – Conclusion

Two general models of realistic mesoscopic solid-state detectors have been described in this paper. The detectors are based on ability of the measured system to control transport current between two particle reservoirs. The models enable detailed analysis of the dynamics of the measurement process. Wavefunction reduction is introduced in this dynamics through the assumption of suppressed quantum coherence between the particle states in different reservoirs. This procedure is very natural and can be justified qualitatively within the general approach to decoherence in quantum systems. The main element of the justification is the increased level of difficulty of maintaining quantum coherence between the states of progressively more complex systems. This fact makes the boundary between the quantum and classical domains not very sharp and dependent on details of specific measurement set-up. This leads to an interesting question whether it is possible to formulate more general and self-consistent conditions defining the boundary between the quantum and classical behaviors of dynamic systems. Mesoscopic solid-state structures provide a convenient setting for further studies of this question.

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Part of the discussion in this paper is based on the work done in collaboration with A. di Lorenzo, A.N. Korotkov, K. Rabenstein, R. Ruskov, V.K. Semenov, E.V. Sukhorukov, and W. Mao. The author would like to thank them, and also G. Benenti, R. Fazio, J.W. Lee, F. Plastina, and D.L. Shepelyansky, for collaboration and discussions of quantum measurements. This work was supported by the NSF under grant # DMR-0325551.

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